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Dynamic Response-Based Characterization of Ring-Based Vibratory Angular Rate Sensors

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Abstract

Dynamic response behaviour of a rotating ring is investigated in order to better understand the achievable performance improvements as well as system limitations. For this purpose, the governing equations that represent the transverse as well as the tangential in-plane motion of a rotating ring are derived via the Hamilton's principle. These equations are then discretized to represent a two-degree-of-freedom time-varying gyroscopic system. The asymmetry effects are considered important and are included by considering mass mismatch in the system mass matrix. In order to predict dynamic behaviour of a ring system subjected to external excitation and body rotation, time and frequency response analyses are performed. The natural frequency variations due to the gyroscopic coupling presented in the system are first characterized for varying input angular rates. The effects of system parameters such as damping and mass mismatch on the sensor sensitivity and operating range are quantified via suitable time and frequency response analyses.

Keywords: Rotating ring; Vibratory angular rate sensor

1. Introduction

Vibratory angular rate sensors have received noticeable attention, in the recent past, primarily due to the economical and technological advantages that this class of sensors offers. In many practical applications, this class of sensors can potentially replace the conventional angular rate sensors such as mechanical and optical gyroscopes, which are typically large and expensive. These sensors that use vibrating structures to provide gyroscopic torque from Coriolis acceleration have the potential for mass production via micromachining processes thereby resulting in significant cost reductions. Among this class of sensors, ring-type structures have gained much acceptance due to inherent advantages such as minimal drift to temperature fluctuation, high sensitivity to rotation, and less sensitivity to environment vibrations (Maluf, 2000). To date, the performance level of these sensors has barely achieved the rate-grade that is considered as the lowest achievable performance grade. Thus, efforts to improve the performance level of micromachined vibratory angular rate sensors are receiving significant attention at present. However, considerations of the dynamic behaviour of this class of sensors have received only limited attention. A rigorous dynamic analysis that focuses on dynamic response is considered important in order to gain better understanding of achievable performance improvements that are possible for this class of rate sensors, and the present paper attempts to fill this void.

Bickford and Reddy (1985) investigated problems concerned with in-plane vibrations of a rotating ring. The effects of extensional and shear deformation and of rotary inertia on the natural frequency variations

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were performed. Huang and Soedel (1987) investigated the in-plane vibration behaviour of rotating rings. They performed a detailed study on the influence of rotational speed and an elastic support on the natural frequencies and mode shapes. Eley *et al.* (2000) considered the Coriolis coupling effect between the in-plane and the out-of-plane motions with the intent of applying their findings to the design of dual axis angular rate sensors. In the present study, dynamic response-based characterization of a ring System subjected to external excitation and body rotation is performed with respect to parameters such as damping, input angular rate and the ring asymmetry.

For this purpose, an approximate form of the equations of motion is first obtained from a rotating ring model developed by Huang and Soedel (1987). The input angular rate is considered time varying, and as a result, the centrifugal terms appear explicitly in the normalized equations of motion. Time and frequency response analyses are performed using the obtained governing equations in order to predict dynamic behaviour of the ring system subjected to external excitation and body rotation. The natural frequency variations due to the gyroscopic coupling present in the system are first characterized for varying input angular rates. The effects of system parameters such as damping and mass mismatch on the sensor sensitivity and operating range are quantified via suitable time and frequency response analyses.

2. Equations of motion

The ring used for the present study is assumed to possess isotropic and homogeneous material properties, and the transverse shear deformation effects are considered negligible in accordance with the thin ring assumption. Figure 1 illustrates the ring supported internally with eight springs. A body fixed frame, $X-Y-Z$, has been used for representing the angular motion of the ring with respect to the inertial frame 'R', and the locations of the neutral surface elements in the rotational coordinates can be defined by introducing curvilinear surface coordinates. In the figure, r represents the mean radius of the ring.

Considering that the ring is rotating about the Z-axis with an angular rate, $\Omega(t)$, the equations that govern the in-plane motion of the ring can be derived using the Hamilton's principle. The rotational rate $\Omega(t)$ is assumed to be time-dependent in this

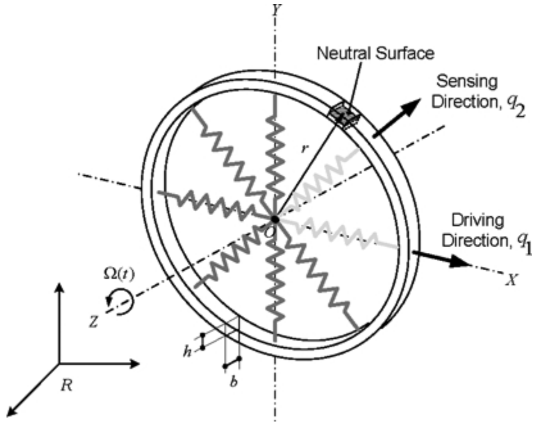

Fig. 1. Schematic of a rotating ring in with support springs.

study, and as a result the equations of motion include terms that contain the angular acceleration term $\dot{\Omega}$. The second flexural modes are chosen for investigating natural frequency variation with the input angular rate. It is known that these resonant modes are generally adopted for typical angular rate sensor applications (Putty and Najafi, 1994). The general expression for the discretized equation can be written in terms of the generalized coordinate vector $\mathbf{q} = [q_1\ q_2]^T$

$$\mathbf{M}\ddot{\mathbf{q}} + \mathbf{G}\dot{\mathbf{q}} + \mathbf{K}\mathbf{q} = \mathbf{0}, \tag{1}$$

and the system matrices can be derived as follows:

$$\mathbf{M} = \begin{bmatrix} 1 & 0 \\ 0 & 1+\delta m \end{bmatrix}, \quad \mathbf{G} = \begin{bmatrix} 0 & -2\Omega\gamma \\ 2\Omega\gamma & 0 \end{bmatrix}, \tag{2}$$

$$\mathbf{K} = \begin{bmatrix} \kappa_1 + \kappa_2\Omega^2 & -\dot{\Omega}\gamma \\ \dot{\Omega}\gamma & \kappa_1 + \kappa_2\Omega^2 \end{bmatrix}, \tag{3}$$

where $\mathbf{M}$ is the mass matrix in which a mass mismatch term δm is added to represent the ring asymmetry, $\mathbf{G}$ is the skew-symmetric gyroscopic matrix which results from Coriolis acceleration and $\mathbf{K}$ is the stiffness matrix. The approximated parameters γ, κ_1 and κ_2 are constant values that depend on the considered mode number and the physical properties of a ring. The details on the system matrices can be found in the paper by Cho and Asokanthan (2005), and further details of derivation of the equations can be found in the paper by Huang and Soedel (1987). The support springs are considered to possess significantly low stiffness and hence assumed not to have significant effects on the ring dynamics.

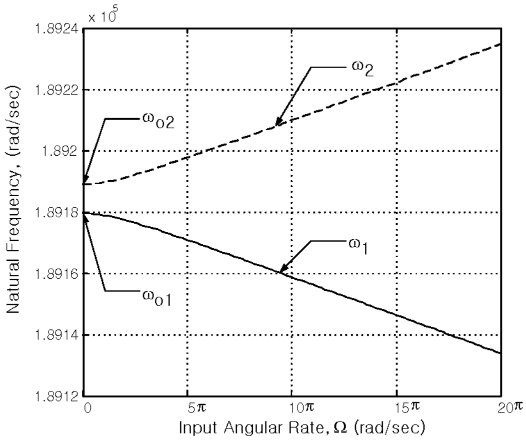

Fig. 2. Natural frequency variations

3. Dynamic response analysis

3.1 Natural frequency variation

Owing to the speed-dependent gyroscopic coupling and system stiffness, it is known that bifurcations of natural frequencies can take place. The variations of the corresponding natural frequencies with input angular rate are depicted in Fig. 2. As a result of an assumed mass mismatch, it can be observed that the natural frequencies associated with a non-rotating ring system are not identical.

3.2 Time response

The response of the ring associated with the generalized coordinate q_2 (sensing direction), is investigated when the ring is resonated in its second degenerate mode pair via excitation provided along the driving generalized coordinate q_2 while the sensor is subjected to an input angular rate. The energy transfer between two degenerate modes takes place via the inherent gyroscopic coupling present in the system. In practice, this condition is employed in typical angular rate sensors and hence both the time response and the frequency response predictions have direct significance to the operation of these devices. For the purpose of representing a suitable profile for the input, the input angular rate is assumed to start from a zero value and reach a steady-state angular speed via a smooth increase in speed. When the input angular motion and the harmonic excitation are considered simultaneously, the time responses of the ring in the sensing direction is depicted in Fig. 3(a) for the two different steady-state angular speeds. The

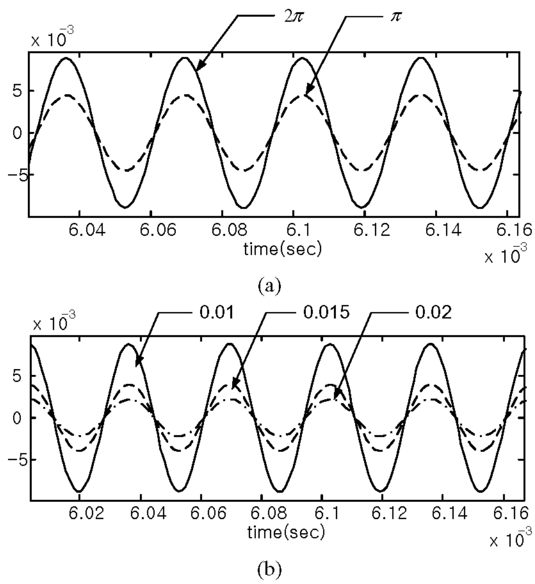

Fig. 3. Variation of radial displacement for different. (a) input angular rates (b) damping ratios

response in the sensing direction takes the modulated form of two signals, i.e., signal from the input angular motion is modulated by the signal from the high frequency external excitation. This figure shows the response, when suitably demodulated, for harmonic angular rate fluctuations with amplitudes π and 2π . As expected, for higher input angular rates, larger amplitude of response is obtained meaning that a larger angular-shift between two degenerate modes takes place.

Figure 3(b) shows the effects of damping values on the time response. The response amplitudes are seen to become smaller as damping is increased. It is worth pointing out that smaller amplitudes of response in the sensing direction will result in less sensitivity for the sensors since the sensor measures the angular rate by measuring the radial displacement of the ring in the sensing direction.

The mass mismatch in the ring has been identified as one of the important parameters that affect the system dynamics. Figure 4 illustrates the response amplitudes for the ring in the sensing direction.

The response in the sensing direction becomes smaller as the mass mismatch between the two normal modes of the rotating ring is increased. Hence, the presence of mass mismatch in the ring-type vibratory angular rate sensor will result in lower sensitivity for the angular rate sensor. In addition, it is also seen that an increase in mass mismatch appears to result in a phase shift in the response.

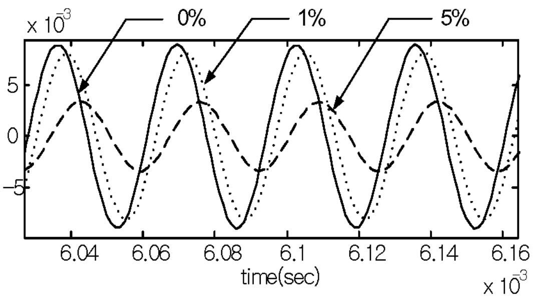

Fig. 4. Variation of radial displacement for different mass-mismatch values.

3.3 Frequency response

The amplitude ratio and the magnitude of frequency response are observed while the ring is excited with the non-rotating ring natural frequency, ω_{o1}, associated with the generalized coordinate q_1. The magnitude variation of frequency response is depicted in Fig. 5(a), where Q_2, f_1 represent, respectively, the response and the input rate amplitudes. As the input angular rate increases, the magnitude of frequency response, although indicates a linear trend for lower angular rates, starts becoming non-linear for higher angular rates as illustrated in the figure. Since the displacement of the ring in the sensing direction (i.e., associated with the generalized coordinate q_2) is used for measuring the angular rate that a sensor is subjected to, this non-linear variation may limit the measurement range of ring-type vibratory angular rate sensors.

The influence of damping on the frequency response is examined, giving particular importance to resonance peaks. Figure 5(b) illustrates that when the damping increases, the magnitudes of the frequency response decrease. However, increased damping results in increased linear range as illustrated in the figure. On the other hand, lower damping for the sensor results in higher sensitivity as seen from the figure. For instance, a larger magnitude range for the same input range can be obtained for sensors with lower damping.

Effects of mass mismatch on the amplitude ratio and frequency response are examined next. As the mass mismatch of the ring increases, the magnitudes of the frequency response decrease as illustrated in Fig. 6. A significant magnitude difference between a symmetric and an asymmetric ring is illustrated in the figure. Hence, the highest sensitivity for the ring-type vibratory angular rate sensor may be obtained when

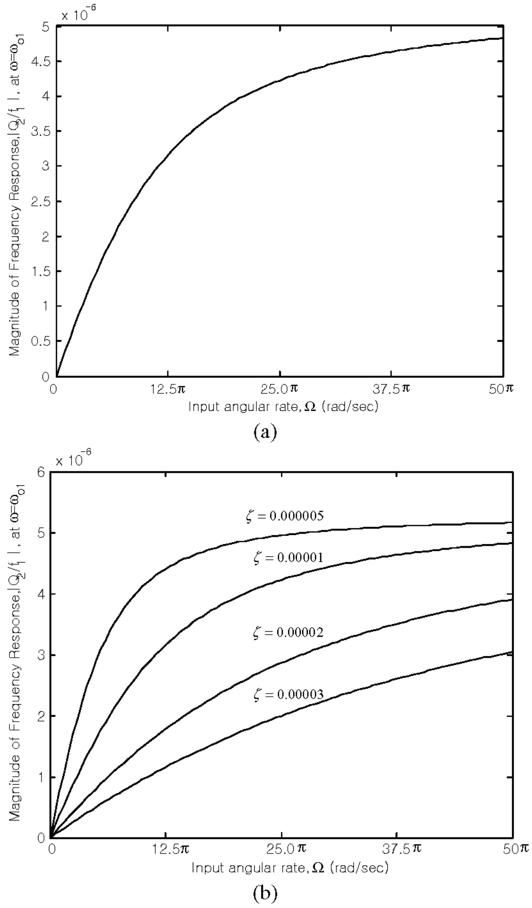

Fig. 5. (a) Magnitude of frequency response at $\omega = \omega_{o1}$ with 0.01% of mass mismatch (b) Variation of frequency response for different damping ratios.

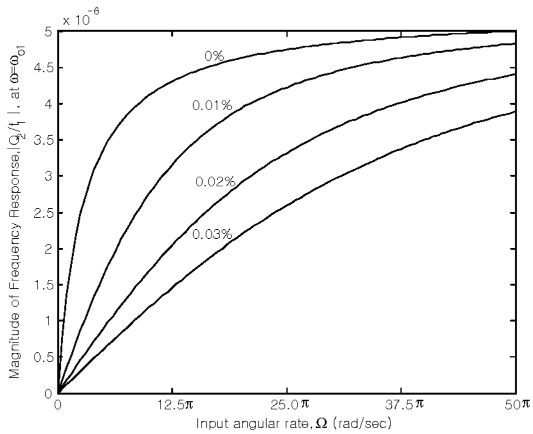

Fig. 6. Variation of frequency response for different mass mismatch.

there is no mass mismatch in the ring. This prediction also agrees with that predicted using the time response. This feature has been exploited in the typical ring-type vibratory angular rate sensors that are currently available. However, it must be pointed out that reduced (or zero) mass mismatch can lead to reduced operating range for the sensor.

4. Conclusions

An analysis is performed for investigating the dynamic response characteristics of a ring element when it is used as part of an angular rate sensor. When the ring is subjected to an input angular rate, the natural frequency variations caused by gyroscopic coupling in the system matrix are investigated. Time and frequency responses of the ring are examined when the ring is excited by a harmonic external driving force while the sensor is subjected to an angular rate. Response amplitudes are obtained when parameters such as the input angular rate, damping, and mass mismatch are varied. It is found that the presence of damping and mass mismatch in the sensors reduces the sensor sensitivity. However, it appears to increase the operating range of the sensors. The prediction from this section can serve as a guide for ring-type vibratory angular rate sensor designs.

Acknowledgements

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